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**A Theory of Microwave Propulsion for Spacecraft**

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## Abstract

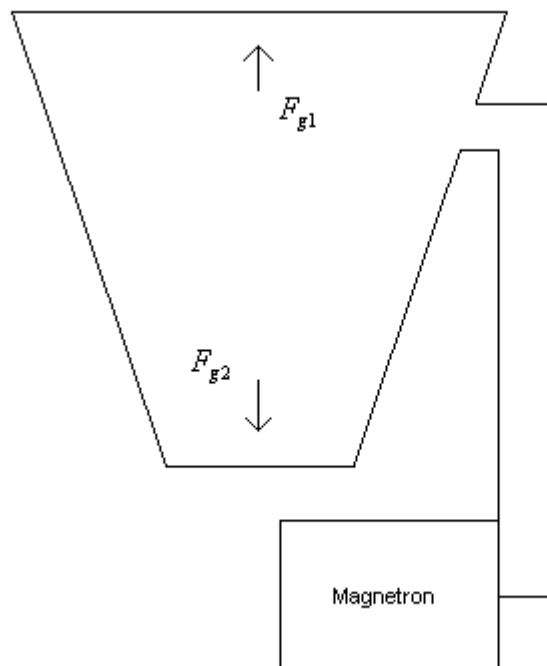
A new principle of electric propulsion for spacecraft is introduced, using microwave technology to achieve direct conversion of d.c. power to thrust without the need for propellant.

A simplified illustrative description of the principles of operation is given, followed by the derivation, from first principles, of an equation for the thrust from such a device. The implications of the law of conservation of energy are examined for both static and dynamic operation of the device.

## 1. Basic Principles

The technique described in this paper uses radiation pressure, at microwave frequencies, in an engine which provides direct conversion from microwave energy to thrust, without the need for propellant.

The concept of the microwave engine is illustrated in fig 1. Microwave energy is fed from a magnetron, via a tuned feed to a closed, tapered waveguide, whose overall electrical length gives resonance at the operating frequency of the magnetron.



The group velocity of the electromagnetic wave at the end plate of the larger section is higher than the group velocity at the end plate of the smaller section. Thus the radiation pressure at the larger end plate is higher than that at the smaller end plate. The resulting force difference ( $F_{g1} - F_{g2}$ ) is multiplied by the  $Q$  of the resonant assembly.

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This force difference is supported by inspection of the classical Lorentz force equation (reference 1).

$$F = q(E + vB) . \quad (1)$$

If  $v$  is replaced with the group velocity  $v_g$  of the electromagnetic wave, then equation 1 illustrates that if  $v_{g1}$  is greater than  $v_{g2}$ , then  $F_{g1}$  should be expected to be greater than  $F_{g2}$ .

However as the velocities at each end of the waveguide are significant fractions of the speed of light, a derivation of the force difference equation invokes the difference in velocities and therefore must take account of the special theory of relativity.

Relativity theory implies that the electromagnetic wave and the waveguide assembly form an open system. Thus the force difference results in a thrust which acts on the waveguide assembly.

## **2. Derivation of the thrust equation.**

Consider a beam of photons incident upon a flat plate perpendicular to the beam. Let the beam have a cross-sectional area  $A$  and suppose that it consists of  $n$  photons per unit volume. Each photon has energy  $hf$  and travels with velocity  $c$ , where  $h$  is Planck's constant and  $f$  is the frequency. The power in the incident beam is then

$$P_0 = nhfAc . \quad (2)$$

The momentum of each photon is  $hf/c$  so that the rate of change of momentum of the beam at the plate (assuming total reflection) is  $2nhfA$ . Equating this change of momentum to the force  $F_0$  exerted on the plate, we find

$$F_0 = 2nhfA = \frac{2P_0}{c} , \quad (3)$$

which is the classical result for the radiation pressure obtained by Maxwell (reference 2). The derivation given here is based on Cullen (reference 3). If the velocity of the beam is  $v$  then the rate of change of momentum at the plate is  $2nhfA(v/c)$ , so that the force  $F_g$  on the plate is in this case given by

$$F_g = \frac{2P_0}{c}(v/c) . \quad (4)$$

We now suppose that the beam enters a vacuum-filled waveguide. The waveguide tapers from free-space propagation, with wavelength  $\lambda_0$ , to dimensions that

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give a waveguide wavelength of  $\lambda_g$  and propagation velocity  $v_g$ . This is the group velocity and is given by

$$v_g = \frac{c}{\sqrt{\mu_r e_r}} \frac{\lambda_0}{\lambda_g}. \quad (5)$$

Then from (4) and (5) (with  $\mu_r = e_r = 1$ ) the force on the plate closing the end of the waveguide is

$$F_g = \frac{2P_0}{c} (v_g / c) = \frac{2P_0}{c} \frac{\lambda_0}{\lambda_g}; \quad (6)$$

see Cullen (p.102 Eq. (15)).

Assume that the beam is propagated in a vacuum-filled tapered waveguide with reflecting plates at each end. Let the guide wavelength at the end of the largest cross-section be  $\lambda_{g1}$  and that at the smallest cross-section be  $\lambda_{g2}$ . Then application of (6) to each plate yields the forces

$$F_{g1} = \frac{2P_0}{c} \frac{\lambda_0}{\lambda_{g1}}, \quad F_{g2} = \frac{2P_0}{c} \frac{\lambda_0}{\lambda_{g2}}.$$

Now  $\lambda_{g2} > \lambda_{g1}$ , due to the difference in cross-section, and hence  $F_{g1} > F_{g2}$ . Therefore the resultant thrust  $T$  will be

$$T = F_{g1} - F_{g2} = \frac{2P_0}{c} \left( \frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}} \right). \quad (7)$$

We note that if the forces had been the mechanical result of a working fluid within the closed waveguide assembly, then the resultant force would merely introduce a mechanical strain in the waveguide walls. This would be the result of a closed system of waveguide and working fluid.

In the present system the working fluid is replaced by an electromagnetic wave propagating close to the speed of light and Newtonian mechanics must be replaced with the special theory of relativity. There are two effects to be considered in the application of the special theory of relativity to the waveguide. The first effect is that as the two forces  $F_{g1}$  and  $F_{g2}$  are dependent upon the velocities  $v_{g1}$  and  $v_{g2}$ , the thrust  $T$  should be calculated according to Einstein's law of addition of velocities given by

$$v = \frac{v_1 + v_2}{1 + (v_1 v_2) / c^2}.$$

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The second effect is that as the beam velocities are not directly dependent on any velocity of the waveguide, the beam and waveguide form an open system. Thus the reactions at the end plates are not constrained within a closed system of waveguide and beam but are reactions between waveguide and beam, each operating within its own reference frame, in an open system.

From (7) and (5) we find

$$T = \frac{2P_0}{c} \left( \frac{v_{g1}}{c} - \frac{v_{g2}}{c} \right),$$

where

$$v_{g1} = c\lambda_0 / \lambda_{g1}, \quad v_{g2} = c\lambda_0 / \lambda_{g2}.$$

Applying the above addition law of relativistic velocities we obtain

$$T = \frac{2P_0}{c^2} \left( \frac{v_{g1} - v_{g2}}{1 - v_{g1}v_{g2}/c^2} \right) = \frac{2P_0 S_0}{c} \left( \frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}} \right), \quad (8)$$

where the correction factor  $S_0$  is

$$S_0 = \left\{ 1 - \frac{\lambda_0^2}{\lambda_{g1}\lambda_{g2}} \right\}^{-1}.$$

The concept of the beam and waveguide as an open system can be illustrated by increasing the velocity of the waveguide in the direction of the thrust, until a significant fraction of the speed of light is reached. Let  $v_w$  be the velocity of the waveguide. Then as each plate is moving with velocity  $v_w$ , the forces on the plates, given by equation 6, are modified as follows:

$$F_{g1} = \frac{2P_0}{c^2} \left( \frac{v_{g1} - v_w}{1 - v_{g1}v_w/c^2} \right) = \frac{2P_0}{c^2} v_{ga}$$

and

$$F_{g2} = \frac{2P_0}{c^2} \left( \frac{v_{g2} + v_w}{1 + v_{g2}v_w/c^2} \right) = \frac{2P_0}{c^2} v_{gb}$$

The thrust is then given by

$$T = \frac{2P_0}{c^2} \left( \frac{v_{ga} - v_{gb}}{1 - v_{ga}v_{gb}/c^2} \right) \quad (9)$$

Thus as the velocity of the waveguide increases in the direction of thrust, the thrust will decrease until a limiting velocity is reached when  $T=0$ . This limiting value of  $v_w$  is reached when  $v_{ga} = v_{gb}$ . Fig 2 illustrates the solution to equation 9 for values of  $v_w$

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from 0 to  $c$ , where  $v_{g1} = 0.95c$ ,  $v_{g2} = 0.05c$ . It can be seen that if  $v_w$  is increased beyond the limiting value of  $0.7118c$ , the thrust reverses.

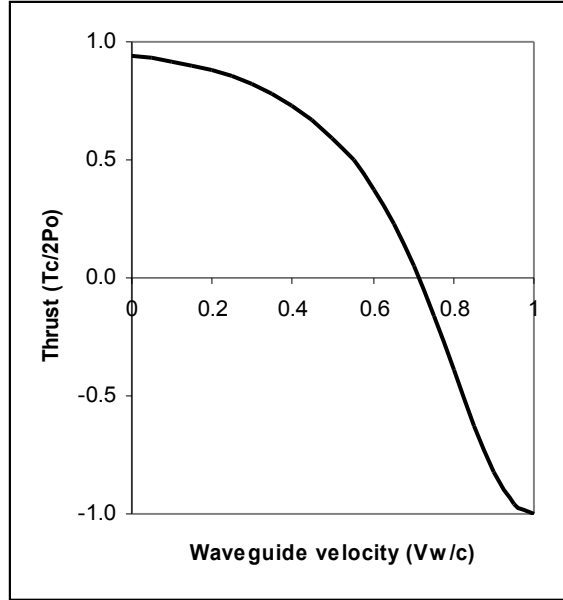


Fig 2. Solution to equation 9.

Returning to a stationary waveguide, we now let the waveguide include a dielectric-filled section at the smaller end of the taper and choose the dimensions to ensure a reflection-free transmission of the beam from the vacuum-filled section to the dielectric-filled section. Note that the reflection-free interface, with matched wave impedances, will ensure no forces are produced at the interface. We also suppose that the dielectric medium has  $\mu_r = 1$ . Then the velocity  $v_{g2}$  is replaced by  $v_{g3}$

$$v_{g3} = c \frac{\lambda_d}{\sqrt{\epsilon_r} \lambda_{g3}}$$

where  $\lambda_d$  is the wavelength in the unbounded dielectric medium and  $\lambda_{g3}$  is the guide wavelength at the end plate of the dielectric section. It then follows that the thrust takes the form

$$T = \frac{2P_0 S_d}{c} \left( \frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{\epsilon_r} \lambda_{g3}} \right) \quad (10)$$

where

$$S_d = \left\{ 1 - \frac{\lambda_0 \lambda_d}{\sqrt{\epsilon_r} \lambda_{g1} \lambda_{g3}} \right\}^{-1}$$

We suppose that the composite waveguide is resonant at the frequency of the microwave beam and that the conductive and dielectric losses are such that there are  $Q$  return paths (each at power  $P_0$ ). Then the total thrust is finally given by

$$T = \frac{2P_0QS_d}{c} \left( \frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{\epsilon_r}\lambda_{g3}} \right) \quad (11)$$

### 3. Dynamic operation of the engine.

We now examine the implications of the principle of the conservation of energy when the thrust is first measured on a static test rig, and then when an engine is used to accelerate a spacecraft.

With the microwave engine mounted on a static test rig all the input power  $P_0$  is converted to electrical loss. In this case the  $Q$  of the engine may be termed  $Q_u$ , the unloaded  $Q$ .

Now 
$$Q_u = \frac{P_c}{P_0} = \frac{P_c}{P_e},$$

where  $P_c$  is the circulating power within the resonant waveguide and  $P_e$  is the electrical loss. From (11) we find

$$T = \frac{2P_0D_fQ_u}{c},$$

Where  $D_f$  is the design factor

$$D_f = S_d \left( \frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{\epsilon_r}\lambda_{g3}} \right).$$

Then 
$$T = \frac{2D_fP_c}{c}. \quad (12)$$

Thus if the circulating power remains constant, for instance in a superconducting resonant waveguide, then  $T$  will remain constant. This will be important in non spacecraft applications where very high values of  $Q_u$  could be employed to provide a constant thrust to counter gravitational force.

If the engine is mounted in a spacecraft of total mass  $M$  and is allowed to accelerate from an initial velocity  $v_i$  to a final velocity  $v_f$  in time  $\Delta t$ , then by equating kinetic energies we obtain:

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$$P_k \Delta t = \frac{M}{2} (v_f^2 - v_i^2)$$

where  $P_k$  is the output power transferred to the spacecraft. From this we obtain

$$P_k \Delta t = \frac{M}{2} (v_f - v_i)(v_f + v_i),$$

so that

$$P_k = M \bar{v} a, \quad (13)$$

where  $\bar{v}$  is the average velocity over time  $\Delta t$  and  $a$  is the acceleration of the spacecraft.

Now  $M.a$  is the force due to the acceleration of the spacecraft, which opposes the thrust of the engine. Then

$$P_k = \frac{2P_0 Q_l D_f \bar{v}}{c} \quad (14)$$

where  $Q_l$  is the loaded  $Q$  of the engine when it is delivering an output power  $P_k$ .

The electrical power losses  $P_e$  are assumed to be  $I^2 R$  losses and thus for any value of  $Q$ ,

$$P_e = Q^2 P_{e0}$$

where  $P_{e0}$  is the loss for  $Q=1$ . From the static case, we have

$$P_{e0} = \frac{P_0}{Q_u^2},$$

so that

$$P_e = P_0 \left( \frac{Q_l}{Q_u} \right)^2. \quad (15)$$

For an accelerating spacecraft,

$$P_0 = P_e + P_k.$$

Substitution of (14) and (15) into this last equation then yields

$$\left( \frac{Q_l}{Q_u} \right)^2 + \frac{2Q_l D_f \bar{v}}{c} = 1. \quad (16)$$

Fig 3.1 shows the solution to (16) for values of  $\bar{v}$  up to 10km/sec and for values of  $Q_u$  equal to  $5 \times 10^3$ ,  $5 \times 10^4$  and  $5 \times 10^5$ . The value of  $D_f$  is taken to be 0.945, resulting from the same group velocities as those used in fig 3.



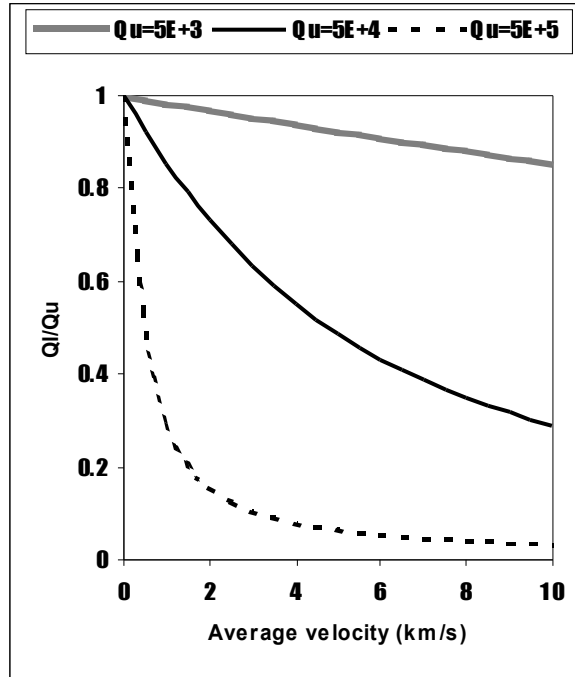


Fig 3.1 Solution to equation 16.

For  $D_f$  equal to 0.945 and an average velocity of 3 km/s, the specific thrust is obtained from (11) and (16) and is given in fig 3.2. This illustrates that the specific thrust increases to a maximum of 333 mN/kW at this velocity.

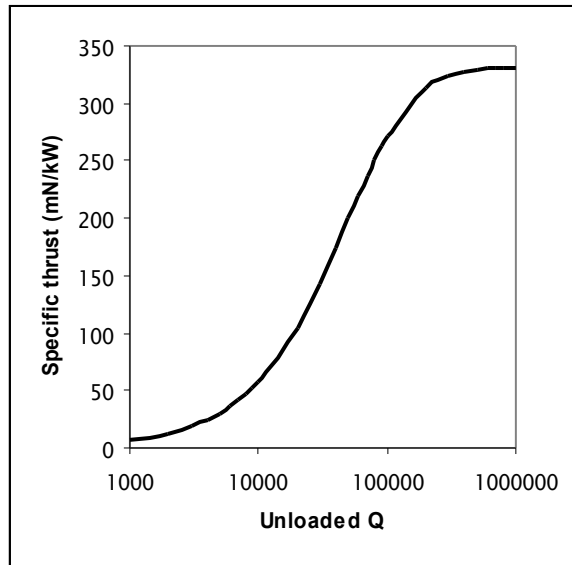


Fig 3.2 Specific thrust at 3km/s.

#### **4. Conclusions**

A theory has been developed for a propulsion technique, which for the first time, allows direct conversion from electrical energy at microwave frequencies to thrust. An expression has been developed from first principles to enable the thrust for such a technique to be calculated. This expression has been verified by the results of two test programmes carried out on an Experimental Thruster and a Demonstrator Engine.

Consideration of the principle of the conservation of energy has led to the derivation of an equation for the loaded Q of an engine used to accelerate a spacecraft at orbital velocities, which allows the prediction of thrust. The spacecraft system advantages in eliminating the need for propellant, together with the predicted specific thrust, offer a major improvement in overall mission performance.

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